EFFECTS OF THE VESSEL WALL ON ELECTROMAGNETIC FLOW MEASUREMENT

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ABSTRACT The theory of the electromagnetic blood flow measuring technique is extended from the well known case (conductive liquid flowing through an insulating tube) to more realistic situations. First the conductivity of the vessel is taken into account, and the electric potentials in both liquid and vessel wall are calculated. The potential difference V between two points on the outside of the vessel and on an axis at right angles to both magnetic field B and the flow v is computed. The comparison is made with the classical flowmeter result $V = 2Ba\bar{v}$ (a = inner radius of vessel, $\bar{v}$ = mean flow velocity). For an average artery, with a ratio of inside diameter to outside diameter of 0.85, the error is found to be in the order of −7 per cent. The blood is assumed to be four times as conductive as the wall tissue. The induced potentials are then calculated in the liquid, in the vessel wall, and in a thin liquid conductive layer surrounding the artery. A film of serous fluid which is likely to exist between a blood vessel and the applied flowmeter sleeve creates an additional shunt. The voltage between the flowmeter electrodes deviates from the expected result by −10 to −15 per cent if the film thickness is 3 per cent of the outside radius of the tube. The evidence is therefore established that flowmeter cuffs should fit the blood vessels accurately to minimize errors.

INTRODUCTION

The Transactions of the Professional Group for Bio-Medical Electronics (1) of December, 1959, describe the state of the art of blood flow measurement. Twelve out of twenty-eight contributions concerned the electromagnetic flowmeter. A magnetic field B is applied transversely to the flow v. A voltage is measured perpendicular to both v and B, between the points A and B (Fig. 1). All the authors assume and demonstrate experimentally that this voltage is proportional to the average flow, that is:

$$\bar{v} = \frac{1}{S}\int_s v\, ds \qquad [1]$$

where S is the cross-sectional area of the vessel. It can even be said that the electromagnetic flowmeter is the only one based on this averaging principle.

Theoretical accounts of the principle have been given by several authors. Williams (2) who was interested in measuring mercury flow calculated the electric field and induced currents for two special velocity distributions: parabolic "Poiseuille" flow and quartic "turbulent" flow. Kolin (3) described an alternating field electromagnetic flowmeter, analyzed its physical principle, and also deduced the correct result for flow in an insulating tube. His result is valid as long as the flow pattern has rotational symmetry. Shercliff (4) gave a more general solution for

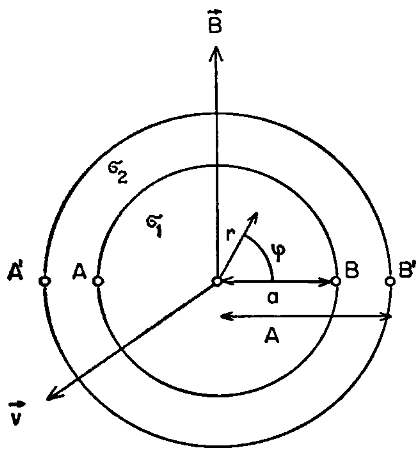

FIGURE 1 *Geometry of the Flowmeter.*
This figure shows the cross-section of the tube through which the liquid is flowing. v is the velocity and σ_1 is the conductivity of the liquid. σ_2 is the conductivity of the vessel wall. The electrodes are located at A' and B'. The magnetic field B is perpendicular to both v and the electrode axis. The voltage between A and B is $-2aB\bar{v}$ only if $\sigma_2 = 0$, where $\bar{v}$ is the average flow velocity. The voltage between A' and B' is always less than $2aB\bar{v}$ for a non-zero σ_2.

flow in an insulating conduit. He calculated that asymmetry in the flow could introduce errors up to a factor of 2. Wyatt (5) recently discussed effects due to nonuniformities of the magnetic field.

Several other authors have considered conductive vessels. Thuerlemann (6) measured mercury and saline flow and proved the voltage independency of the velocity distribution, by tacitly neglecting deviations from symmetry. He gave a rough approximation for the case where the vessel wall conducts much better than the liquid. His result is therefore not biologically useful. Cushing (7), in a thorough treatment, showed all the previous workers' reports correct as long as the conductivity of the flowing liquid is larger than 10^{-7} (ohm meter)$^{-1}$. He also investigated the influence of a conductive wall for the case where the outside potential is kept zero by a perfect conductor. Hill (8) developed simultaneously with and in cooperation with the author the general equations describing the influence of a conductive wall. The correspondence with Professor Hill was extremely stimulating and the author gratefully acknowledges his help.

REVIEW OF THE THEORETICAL BASIS

Maxwell's equations and Ohm's law describe the induced electric fields and currents in a flowing liquid.

First, for an outside observer, both in the liquid and in the wall:

$$\text{Curl } E = -\frac{\delta B}{\delta t} \qquad [2]$$

With a steady magnetic field B, the right-hand side of [2] vanishes. In many applications, however, an alternating field is used. The flowmeter literature (see mainly reference 1) discusses extensively the implications of the induced $\delta B/\delta t$ voltage. To study the influence of the wall we shall consider a steady and uniform field, so that equation [2] becomes

$$\text{Curl } E = 0 \qquad [3]$$

Secondly, we know that

$$\text{div } j = 0 \qquad [4]$$

as long as there are neither free charges nor appreciable polarization currents in the medium (7). Ohm's law in its differential form is

in the liquid: $j_1 = \sigma_1[E_1 + v \times B]$

in the wall: $j_2 = \sigma_2 E_2$

where j = current density, σ = conductivity, and v = velocity of the medium. The potential U used as a measure for the flow is related to E and j by the equations

$$\text{grad } U_1 = -E_1 = -j_1/\sigma_1 + v \times B \qquad [5a]$$

$$\text{grad } U_2 = -E_2 = -j_2/\sigma_2 \qquad [5b]$$

This shows that the potential is not only determined by the electric field due to the motion of the liquid ($v \times B$) but also by the "voltage drop" (j/σ) of the currents flowing because the media are conductive.

There are two ways to find the potential. One is to use equations [3] and [4],

$\text{Curl } E = 0$

rewritten as $\text{Curl } j = \text{Curl } (v \times B)$

and $\text{div } j = 0$

With these relations and the boundary conditions the right-hand side of [5] is completely determined.

As an alternative one can use the potential relation [5] directly. (This was done by all the authors except Williams.) By applying the divergence operator to both sides of [5] one finds

in the liquid: $\nabla^2 U_1 = \text{div } (v \times B)$ [6a]

in the wall: $\nabla^2 U_2 = 0 \quad (v = 0)$ [6b]

The amount of computation is about the same for both methods. Both can be carried through generally, without introducing a particular velocity distribution.

The boundary equations for both procedures are:

Radial currents at an insulating boundary = 0 [7a]

Radial currents at an interface between two media of different conductivity are continuous [7b]

Angular currents at an interface compare inversely as the conductivities (σ_1) and (σ_2) [7c]

([7b] and [7c] together are Snell's law for current lines.)

Current density stays finite [7d]

SHUNTING EFFECT OF A CONDUCTIVE VESSEL WALL

In this section the potentials U_1 and U_2 are calculated for the geometry given in Fig. 1. We assume again that the velocity of the liquid has only an axial (z) component, no angular gradient, and that the magnetic field lies in the direction of the y axis.

Then
$$\nabla^2 U_1 = -Bv_r \cos\varphi$$

where
$$v_r = \frac{\delta v}{\delta r}$$
φ = angle in respect to electrode axis A-B

and in the wall
$$\nabla^2 U_2 = 0$$

These two equations can be integrated in a straightforward manner, because the rotational symmetry allows us to choose solutions of the form of

$$U_1 = W_1(r)\cos\varphi \qquad 0 \leq r \leq a$$
$$U_2 = W_2(r)\cos\varphi \qquad a \leq r \leq A$$

This leads to the following differential equations

$$\left[\frac{W_1}{r}\right]_r + W_{1rr} = -Bv_r$$
$$\left[\frac{W_2}{r}\right]_r + W_{2rr} = 0$$

where
$$W_{1rr} = \frac{\delta^2 W_1}{\delta r^2}, \quad \text{etc.}$$

Integrating twice we get the solutions:

$$W_1 = -B\left[Q(r)\frac{1}{2\pi r} + \frac{K_1}{2}r + \frac{K_2}{r}\right]$$
$$W_2 = \frac{K_3}{2}r + \frac{K_4}{r}$$

where $$Q(r) = 2\pi \int_0^r rv\, dr$$

Now the following boundary conditions are introduced:

$$U_1(0, \varphi) \neq \infty$$

$$\frac{\delta U_2}{\delta r}(A, \varphi) = 0 \qquad \text{(insulating outside)} \qquad [8]$$

$$U_1(a, \varphi) = U_2(a, \varphi) \qquad \text{(continuous potential at the interface between liquid and wall)}$$

$$j\,\mathrm{rad}_1\,(a, \varphi) = j\,\mathrm{rad}_2\,(a, \varphi) \qquad \text{(radial components of current density are continuous)}$$

and since $$v(a) = 0$$

$$\frac{\delta U_1(a, \varphi)}{\delta r} \cdot \sigma_1 = \frac{\delta U_2(a, \varphi)}{\delta r} \cdot \sigma_2$$

These conditions enable us to find the constants of integration

$$K_1 = -\bar{v}(a) \frac{1 - \frac{a^2}{A^2} - \frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right)}{1 - \frac{a^2}{A^2} + \frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right)}$$

$$K_2 = 0$$

$$K_3 = -2B\bar{v}(a) \frac{\frac{\sigma_1}{\sigma_2}\frac{a^2}{A^2}}{1 - \frac{a^2}{A^2} + \frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right)}$$

$$K_4 = \frac{A^2}{2} K_3$$

where $$\bar{v}(a) = \frac{2\pi}{\pi a^2} \int_0^a rv(r)\, dr$$

The potentials therefore become:

in the liquid $$U_1(r, \varphi) = -\frac{B}{2} r[\bar{v}(a) \cdot M + \bar{v}(r)] \cos \varphi \qquad [9]$$

where $$M = \frac{\frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right) - \left(1 - \frac{a^2}{A^2}\right)}{\frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right) + \left(1 - \frac{a^2}{A^2}\right)}$$

$$\bar{v}(r) = \frac{Q(r)}{\pi r^2}$$

in the wall $$U_2(r, \varphi) = -\frac{B}{2}\, a\bar{v}(a)\, N\left(\frac{r}{A} + \frac{A}{r}\right) \cos\varphi \qquad [10]$$

where $$N = \frac{2\frac{\sigma_1}{\sigma_2}\frac{a}{A}}{\frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right) + \left(1 - \frac{a^2}{A^2}\right)}$$

The potential at the electrode ($\varphi = 0$) touching the wall from the outside ($r = A$) becomes

$$U_2(A, 0) = Ba\bar{v}(a)N \qquad [11]$$

Fig. 2 shows a typical potential distribution as it would appear if the wall of the

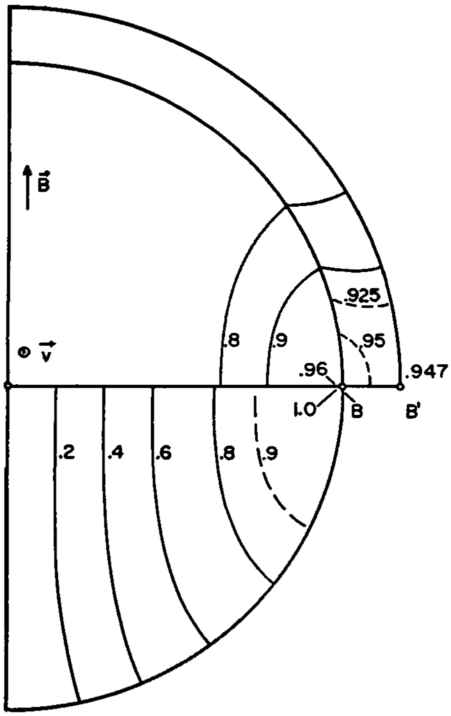

FIGURE 2 *Potential Distribution inside the Liquid and the Vessel for Poiseuille Flow.*

Case I: Bottom Half. The liquid is flowing through an insulating tube ($\sigma_2 = 0$). The magnetic field is applied along the vertical diameter. The isopotential lines are graduated in fractions of ($-2aB\bar{v}$), where $\bar{v}$ is the average flow velocity.

Case II: Top Half. The liquid is surrounded by a tube whose wall is four times less conductive than the liquid. The ratio of inside to outside diameter is 0.85. The outside surrounding is assumed to be an insulator. It is evident that the field is distorted and that the measured voltage is always erroneous. In this particular case the potential at the electrodes is 0.947 of ($-2aB\bar{v}$). The error is —5 per cent.

artery is four times more resistive than blood, and if the velocity profile is a paraboloid (Poiseuille flow).

The potential U_1 becomes identical with the classical flowmeter value for the limiting case of an insulating outside wall ($\sigma_2 = 0$)

since $$\lim_{\sigma_1/\sigma_2 \to \infty} M = 1$$

then $$\lim_{\sigma_1/\sigma_2 \to \infty} U_1 = -\frac{B}{2} r[\bar{v}(a) + \bar{v}(r)] \cos\varphi$$

and at the site of the electrode $(a, 0)$

$$\lim_{\sigma_1/\sigma_2 \to \infty} U_1(a, 0) = -B\bar{v}_{tot}a$$

Therefore the voltage V between the electrodes at A and B becomes

$$V = -2aB\bar{v} \text{ as derived by Kolin and others.} \quad [12]$$

The same value must result for U_2 in the limiting case of zero wall thickness $(a = A)$, thus,

$$\lim_{a/A \to 1} N = 1$$

$$\lim_{a/A \to 1} U_2(a, 0) = -Ba\bar{v}$$

The error e due to the conductive wall can now be defined as

$$e = \frac{U_2(A, 0)}{\lim U_2(a, 0)} - 1 = N - 1. \quad [13]$$

Fig. 3 shows e as a function of the ratio of the outer to the inner diameter of the tube for different values of σ_1/σ_2, the ratio of conductivities.

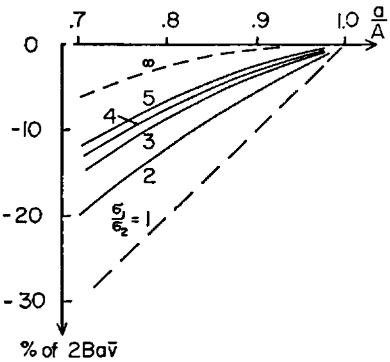

FIGURE 3 *Error in Flow Measurement Introduced by a Conductive Tube.*
Abscissa: Ratio of the inside radius to outside radius of the vessel. Ordinate: Negative error in flow measurement in per cent of the "correct" value $(-2aB\bar{v})$.
The parameter σ_1/σ_2 is the ratio of the conductivities of the liquid and the tube wall.
Typical errors will be in the range of −5 to −7 per cent, since an average a/A ratio is between 0.80 and 0.90, and since σ_1/σ_2 is in the order of 4.

The case of a very thick conductive wall is also of interest:

$$\lim_{a/A \to 0} N = 0 \qquad \lim_{a/A \to 0} e = -1$$

In other words, the error of −100 per cent means that there is no voltage at the *outside* of the tube. At the same time, at the *inside* of the tube $(r = a)$ U_1 becomes:

$$\lim_{a/A \to 0} M = \frac{\sigma_1/\sigma_2 - 1}{\sigma_1/\sigma_2 + 1}$$

$$\lim_{a/A \to 0} U_1(a, 0) = -\frac{B}{2}\bar{v}(a)a\left[1 + \frac{\sigma_1/\sigma_2 - 1}{\sigma_1/\sigma_2 + 1}\right]$$

$$= -Ba\bar{v}(a)\frac{\sigma_1}{\sigma_1 + \sigma_2}$$

A typical σ_1/σ_2 value is 4, according to Schwan (9). (This does not take into account the non-isotropic nature of the wall resistance, but represents an "average" figure.)

Thus for a very thick wall,

$$\lim_{a/A \to 0} U_1(a, 0) = -\tfrac{4}{5} Ba\bar{v}$$

This error of −20 per cent also would be encountered if one could measure the induced voltage intravascularly. (This is equivalent to assuming all of the surrounding tissue four times less conductive than blood.)

An average artery has an a/A ratio of 0.9. A typical error in flow measurement therefore is in the order of −5 to −7 per cent, if one does not take into account the electrical shunt provided by the conductive vessel wall. This result depends only on the ratio of conductivities σ_1/σ_2 and is true as long as σ_1 is larger than 10^{-7} (ohm meter)$^{-1}$ (7). Although only the average flow (equation [1]) appears in the final expression for the measured voltage, it is independent of particular flow profiles only if the flow has rotational symmetry (4).

ERRORS INTRODUCED BY A LIQUID FILM IN BETWEEN THE BLOOD VESSEL AND THE FLOWMETER CUFF

In a real flowmeter situation the errors will be even larger than predicted in the previous section. Any tissue fluid, or blood, which is lying between the vessel and the flowmeter sleeve will provide an additional electric shunt. It was pointed out above that the induced potential is partly due to the $v \times B$ field and partly due to the voltage drop of the current j. A liquid film on the outside of the vessel will provide additional paths for the induced currents. Therefore the potential due to these currents (j/σ integrated) is even more distorted than that due to the vessel wall only.

In this section, the errors caused by this additional shunt are computed. The same method is used as for the previous case. The potential equations are simply extended to a third layer where there is no velocity. The Laplace equation holds in the film as well as in the vessel wall.

The differential equations for the potentials U are

$$\begin{aligned} &\text{in the liquid:} && \nabla^2 U_1 = -Bv_r \cos\varphi \\ &\text{in the wall:} && \nabla^2 U_2 = 0 \\ &\text{in the film:} && \nabla^2 U_3 = 0 \end{aligned} \qquad [14]$$

The boundary conditions analogous to equation [7] are:

$$U_1(0) \neq \infty$$

$$U_1(a, \varphi) = U_2(a, \varphi) \qquad [15]$$

$$\sigma_1 \frac{\delta U_1}{\delta r}(a, \varphi) = \sigma_2 \frac{\delta U_2}{\delta r}(a, \varphi)$$

$$U_2(A, \varphi) = U_3(A, \varphi)$$

$$\sigma_2 \frac{\delta U_2}{\delta r}(A, \varphi) = \sigma_3 \frac{\delta U_3}{\delta r}(A, \varphi)$$

$$\frac{\delta U_3}{\delta r}(A + \Delta A) = 0$$

where a = inner radius of vessel (see Fig. 1)
A = outer radius of vessel,
ΔA = thickness of film,
σ_1 = conductivity of blood,
σ_2 = conductivity of wall,
σ_3 = conductivity of liquid film on the outside.

The solutions are of the following forms after using the first and last of the above mentioned boundary conditions:

$$U_1 = -\frac{B}{2}[\bar{v}(r) + K_1] r \cos\varphi$$

$$U_2 = \left[\frac{K_2}{2} r + K_3 \frac{1}{r}\right] \cos\varphi \qquad [16]$$

$$U_3 = \frac{K_4}{2}\left[r + \frac{A^2}{r}\left(1 + \frac{\Delta A}{A}\right)^2\right] \cos\varphi$$

The four integration constants are found in a straightforward manner by using the rest of the boundary conditions.

Of immediate interest is the expression for U_3, the potential in the liquid film:

$$U_3(r, \varphi) = -Ba\bar{v}(a) \cdot \frac{\left(1 - \frac{\Delta A}{A}\right) \frac{a}{A} \frac{\sigma_1}{\sigma_2} \left[\frac{r}{A} + \frac{A}{r}\left(1 + \frac{\Delta A}{A}\right)^2\right] \cos\varphi}{\frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right) + 1 - \frac{a^2}{A^2} + \left(\frac{\Delta A}{A}\frac{\sigma_3}{\sigma_2}\right)\left[\frac{\sigma_1}{\sigma_2}\left(1 - \frac{a^2}{A^2}\right) + 1 + \frac{a^2}{A^2}\right]}$$

neglecting terms in $\left(\frac{\Delta A}{A}\right)^2$

This expression assumes very closely the same value at $r = A$ as at $r = A + \Delta A$.

$$U_3(A, 0) = -Ba\bar{v}(a) \cdot \frac{2 \frac{a}{A} \frac{\sigma_1}{\sigma_2}}{\frac{\sigma_1}{\sigma_2}\left(1 + \frac{a^2}{A^2}\right) + 1 - \frac{a^2}{A^2} + \left(\frac{\Delta A}{A}\frac{\sigma_3}{\sigma_2}\right)\left[\frac{\sigma_1}{\sigma_2}\left(1 - \frac{a^2}{A^2}\right) + 1 + \frac{a^2}{A^2}\right]}$$

This has the same general form as the potential computed for the simpler case (conductive wall, no film). However, there is an additional error introduced by the term in the denominator

$$\frac{\Delta A}{A}\frac{\sigma_3}{\sigma_2}[1 + a^2/A^2 + \sigma_1/\sigma_2(1 - a^2/A^2)]$$

reflecting directly the influence of the film, with its thickness $\Delta A/A$ and conductivity σ_3.

The total error is plotted in Fig. 4. It is clear that the deviations from the ideal

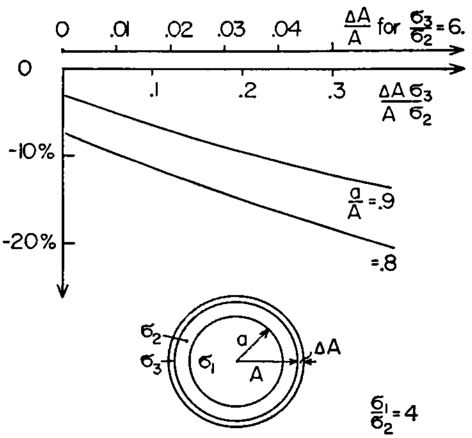

FIGURE 4 *Error in Flow Measurement Introduced by a Conductive Vessel Wall and a Liquid Film "Shunting" the Electrodes.* For this graph a typical σ_1/σ_2 ratio was chosen as 4. The inside diameter to outside diameter ratio is assumed to be 0.8, or 0.9, and the negative errors (in per cent of $-2Ba\bar{v}$) are calculated as a function of the relative film thickness $\Delta A/A$ with a fixed $\sigma_3/\sigma_2 = 6.0$. This means that the film is 1.5 times more conductive than blood (an average value for serous fluids). The other abscissa given shows how the film shunting error depends on the product $\Delta A/A$ σ_3/σ_2. See text for a discussion of a "typical error."

value $(-B\bar{v}a)$ are larger due to the conductive film. An average blood vessel wall alone causes errors between −5 and −7 per cent. If it is surrounded by a liquid film such that

$$\frac{\Delta A}{A}\frac{\sigma_3}{\sigma_2} = 0.016 \times 6.0 \cong 0.1$$

the errors lie between −8 and −10 per cent. This possibly may help to explain the discrepancies still existing between most of the published electromagnetic flow measurements and the ones done by other methods. In particular McDonald (10) with high speed cinematography observed velocities larger than many of the published electromagnetic flowmeter results show (10, p. 123).

A typical film thickness is hard to predict. In any case it is obvious that care should be taken to fit the flowmeter cuff tightly around the blood vessel. Errors occurring will have to be estimated for each cuff.

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REFERENCES

1. PGBME, Transactions of the Professional Group for Bio-Medical Electronics of the IRE, 1959, PGME, No. 4.
2. WILLIAMS, B. J., Induction of electromotive forces in a moving liquid by a magnetic field, and its application to an investigation of the flow in liquids, *Proc. Physic. Soc.* (*London*) 1930, **42**, 466.
3. KOLIN, A., An alternating field induction flow meter of high sensitivity, *Rev. Scient. Instr.* 1945, **16**, 109.
4. SHERCLIFF, J. A., Relation between velocity profile and the sensitivity of the electro-magnetic flowmeter, *J. Appl. Physics,* 1954, **25**, 817.
5. WYATT, D. G., Problems in the measurement of blood flow by magnetic induction, *Physics in Med. and Biol.* 1961, **5**, 289.
6. THUERLEMANN, B., *Helv. Physica Acta,* 1941, **14**, 383.
7. CUSHING, V., Induction flowmeter, *Rev. Scient. Instr.* 1958, **29**, 692.
8. HILL, W. S., On the theoretical basis of el. magn. methods to measure rates of flow, *Bol. Fac. Ingeniera y Agrimensura Montevideo,* 1960, **7**, 1.
9. SCHWAN, H. P., AND KAY, C. F., Specific resistance of body tissue, *Circulation Research* 1956, **4**, 664.
10. McDONALD, D. A., Blood Flow in Arteries, Baltimore, The Williams & Wilkins Co., 1960.